Two complexity measures in economics

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Summary

Some problems are complex

Nash embedding theory $\rightarrow$ embedding space

- isolate short periods of **stability**
- look for **changes** in the stable patterns

Lyapunov exponent $\rightarrow$ a measure of chaos

MLE = maximum Lyapunov exponent

- mean-reverting system has **negative** MLE
- cyclical system has **near zero** MLE
- chaotic system has **positive** MLE

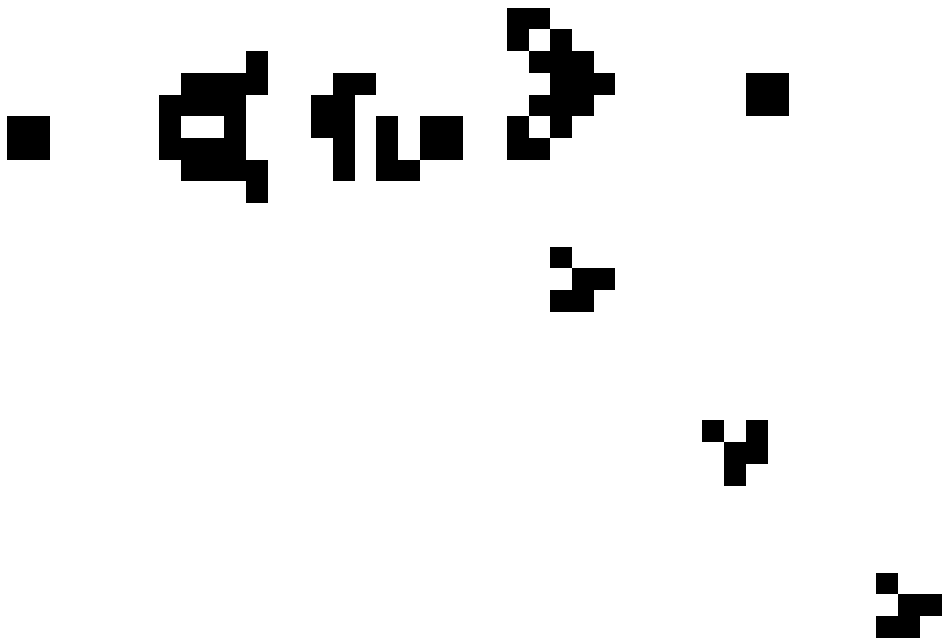
Complexity in economics

It all began with nonlinear models

- Cellular automata and the segregation model (Schelling 1978)
- The El Farol problem (Arthur 1994)
- The auction model contest (Arthur, Holland, LeBaron, Palmer, Tayler 1996)

Cellular automata

Stanislaw Ulam $\rightarrow$ John von Neumann $\rightarrow$ John Conway $\rightarrow$ Game of LIfe
A grid where each cell has state determined from it's own previous state and the current state of its neighbors. (Bill Gosper 1970)



https://www.conwaylife.com/w/images/b/b6/Gosperglidergun.gif

Segregation

Thomas C. Schelling (1969) "Models of segregation", American Economic Review, 1969, 59(2), 488–49
2005 Nobel

El Farol

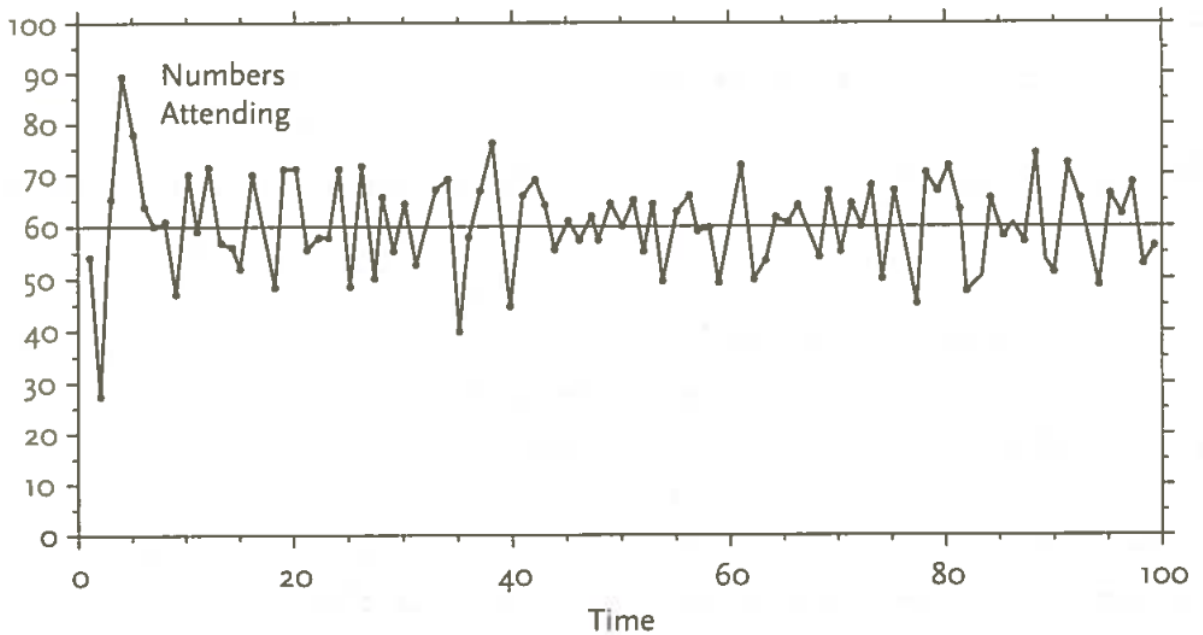
"Inductive reasoning and bounded rationality" - W. Brian Arthur, Santa Fe Institute, AEA, 1994 (Paul Krugman discussant)

- El Farol is a small bar and restaurant on Canyon Road in Santa Fe.
- Brian noticed that sometimes is was too crowded, and sometimes not very crowded, but most of the time the crowd was about the same.
- This is before cellphones, so people weren't texting or facebooking or whatever to coordinate their strategies. No single predictive model could explain this.
- Brian conjectured that each person held a variable number of predictive models and used one until it didn't work, moving on to the next.

El Farol

- Attendance (1,2,3,...) weeks ago,
- Average of last (2,3,4...) weeks
- Trend over last (2,3,4...) weeks
- Reverse trend over last (2,3,4...) weeks

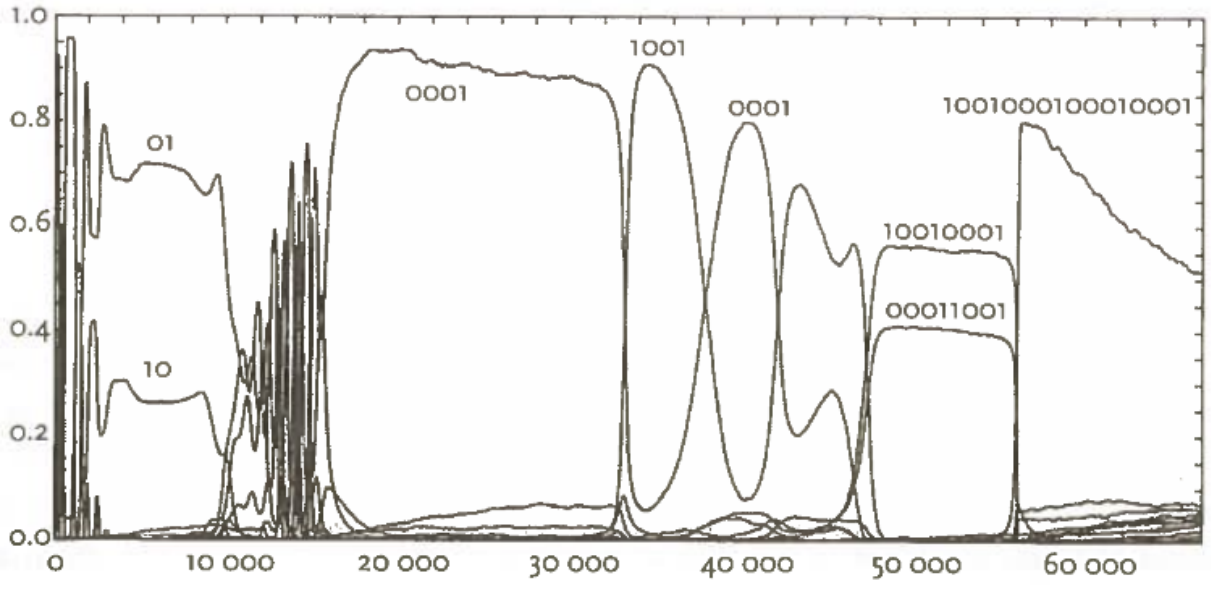
Each individual has some subset of all possible strategies. They switch away from strategy when that one doesn't work. Highly nonlinear.



Auction model

"Asset Pricing Under Endogenous Expectations in an Artificial Stock Market" - Brian Arthur^{E}, John Holland^{CS}, Blake LeBaron^{E}, Richard Palmer^{P}, and Paul Tayler^{CS}, Santa Fe Institute, 1996)

- This was a contest (like Robot Wars) between computer programs.
- Complicated strategies sometimes succeeded for short times, but simple strategies prevailed.

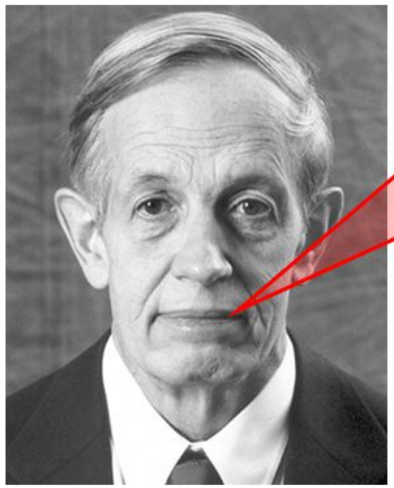


Quantifying complexity

- How do we describe these systems?
- We know complexity when we see it, but how do we quantify it?

Embedding space

Nash embedding theorem



John Forbes Nash

Embedding theorem **(1956): Any smooth Riemannian manifold can be realized as an embedded surface in Euclidean space of sufficiently high yet finite dimension**

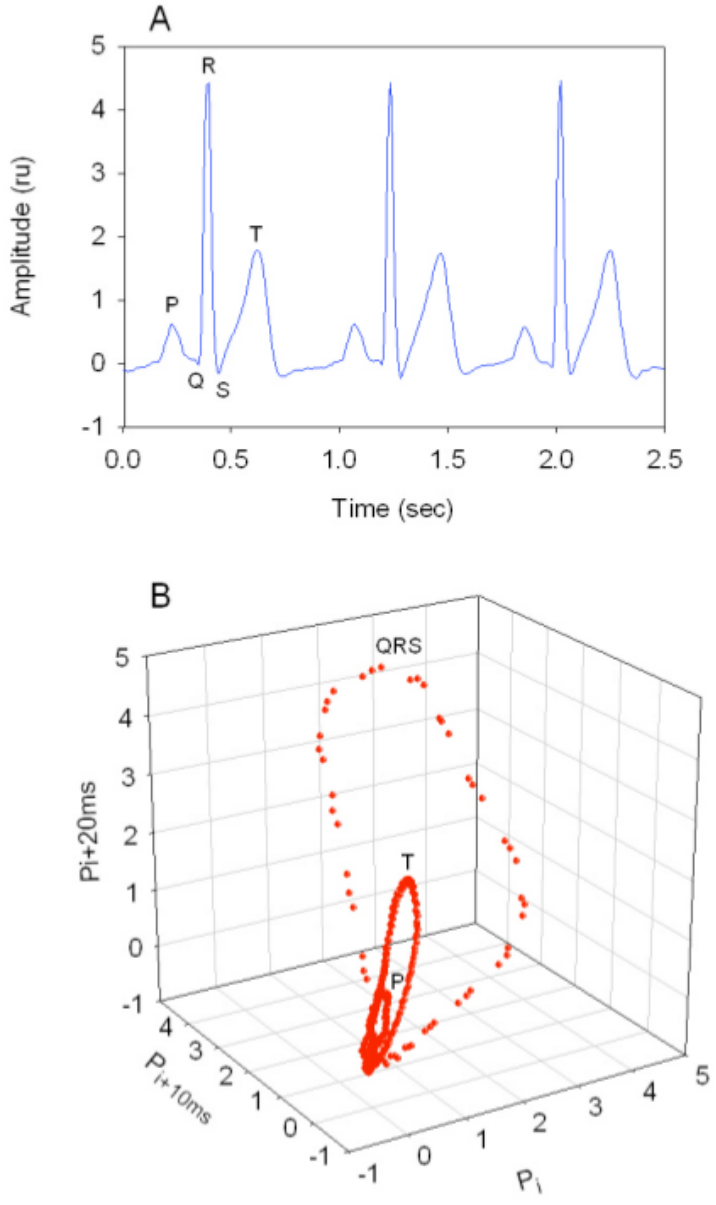
Technical conditions:

- Manifold is $\mathcal{C}^k, k \geq 3$
- For m-dimensional manifold, embedding space dimension is $n = m^2 + 5m + 3$

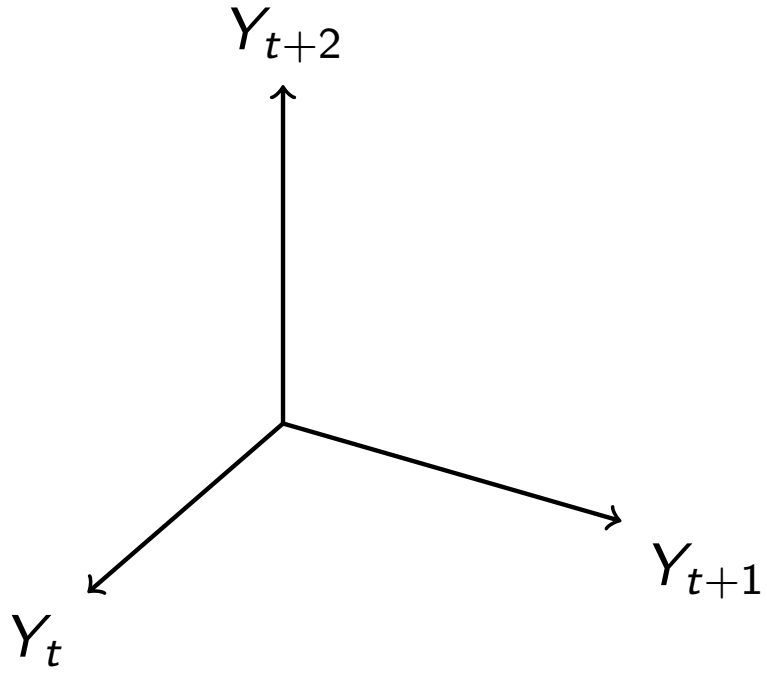
Practically: **intrinsic and extrinsic views are equivalent!**

Nash, 1956

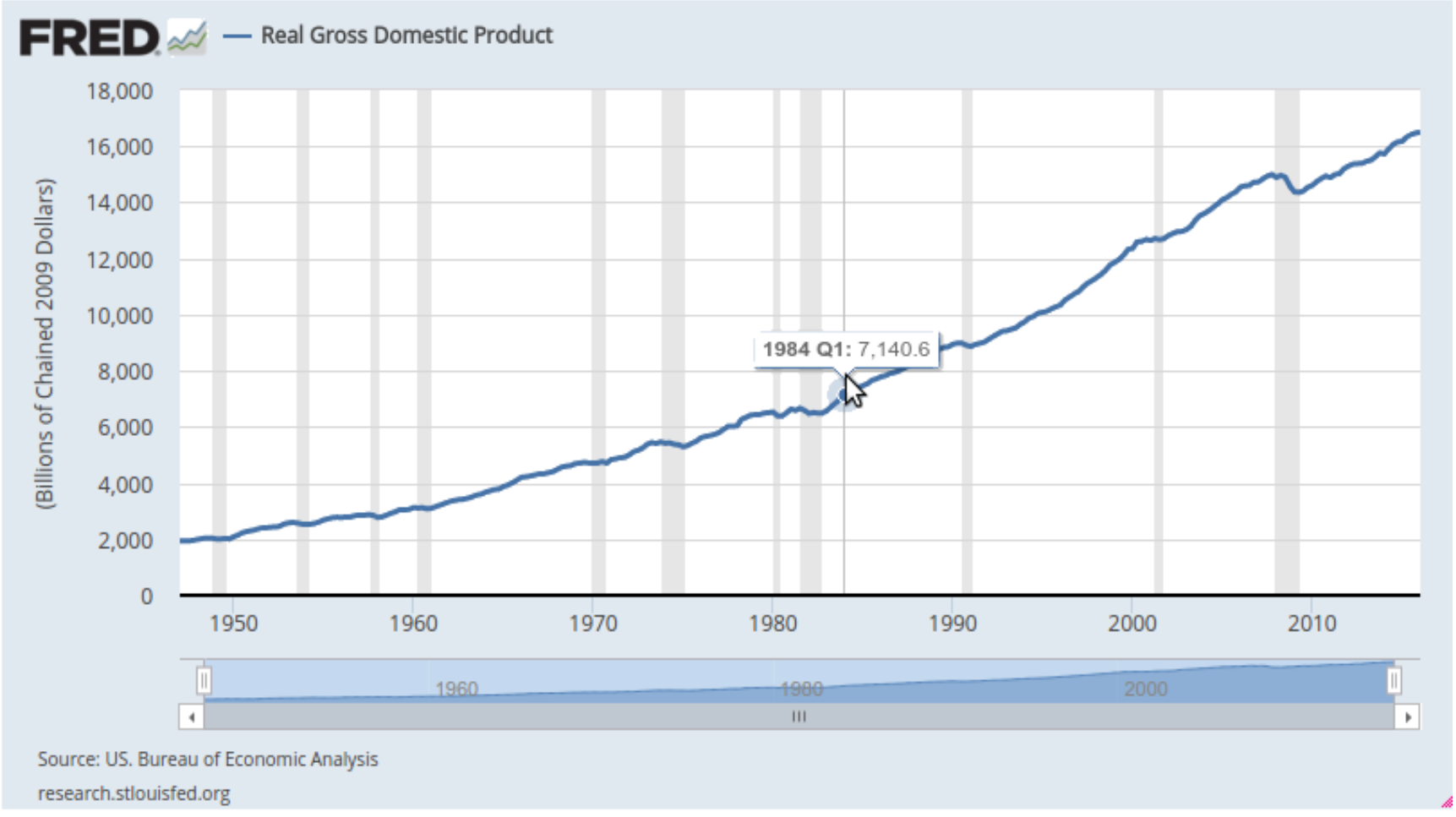
EKG in embedding space



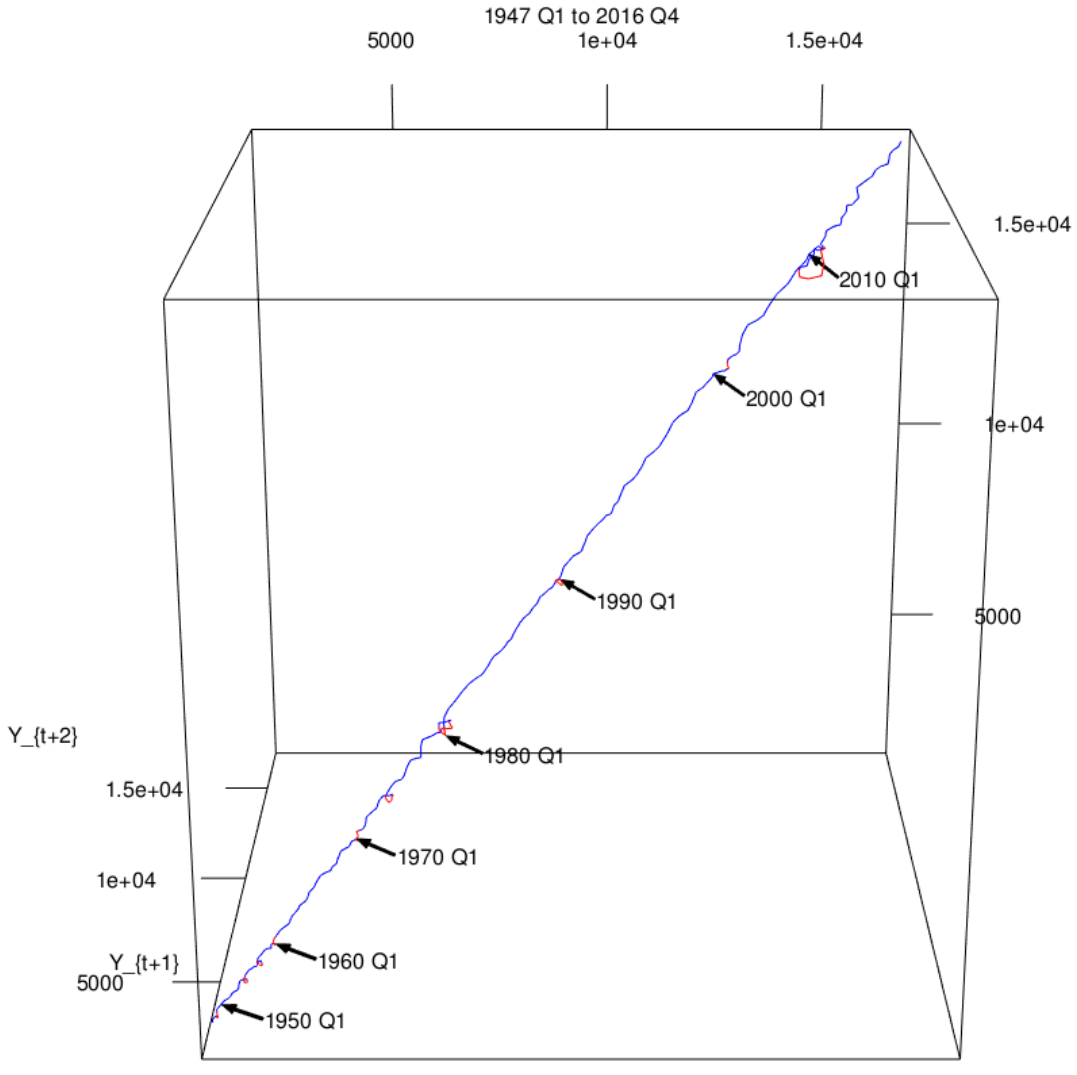
A 3-dimensional embedding space



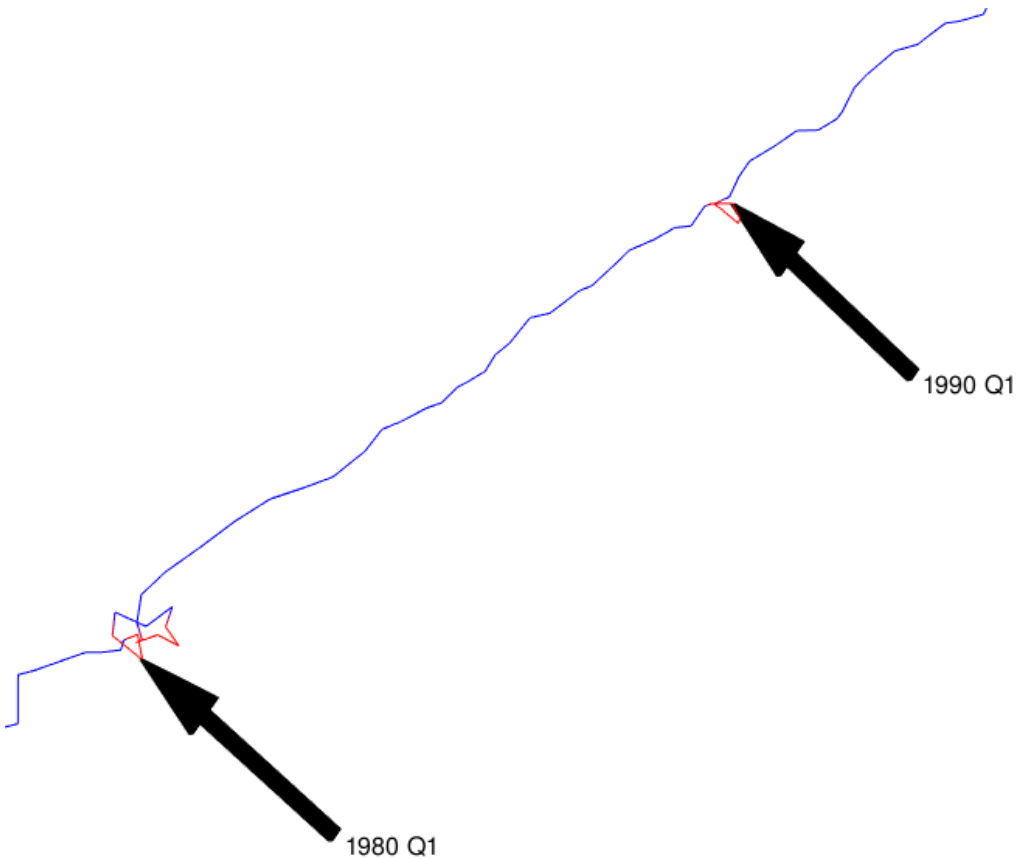
US GDP


FRED
Real Gross Domestic Product
(Billions of Chained 2009 Dollars)
18,000
16,000
14,000
12,000
10,000
8,000
6,000
4,000
2,000
0
1950
1960
1970
1980
1990
2000
2010
1984 Q1: 7,140.6
1960
1980
2000
Source: US. Bureau of Economic Analysis
research.stlouisfed.org

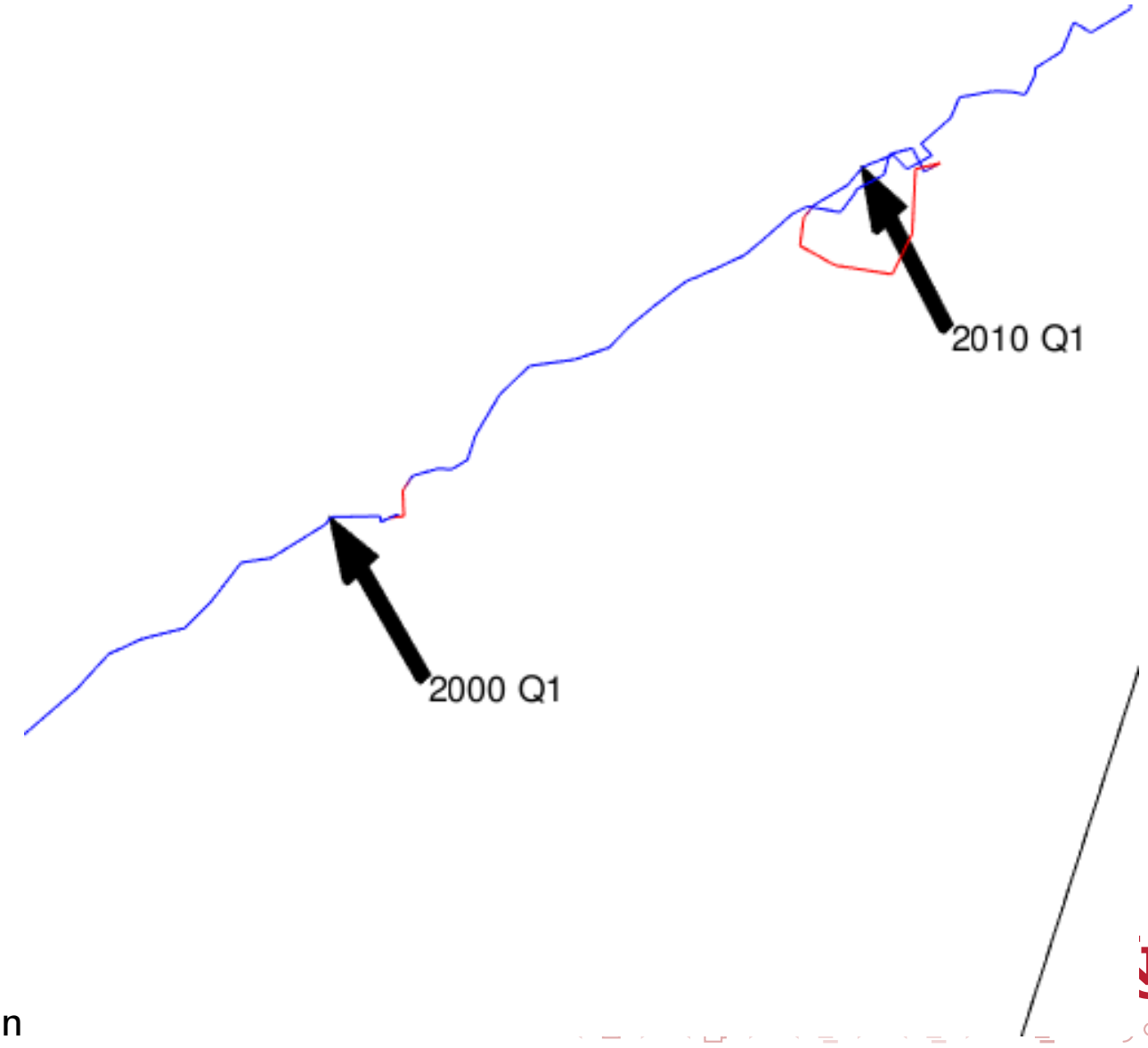
Embedding space



Embedding space



Embedding space

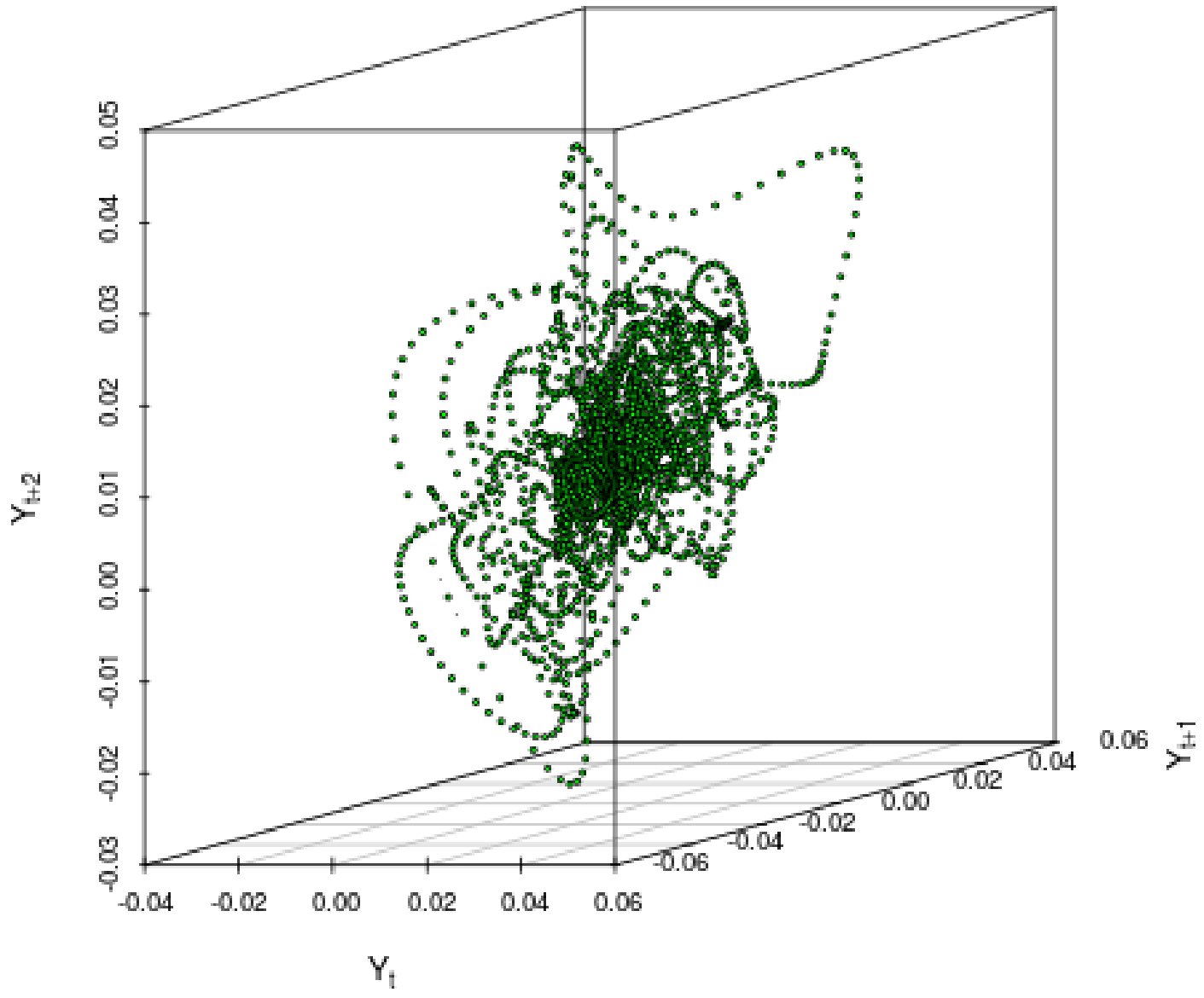


aximum Lyapunov exponen

Embedding space

Interactive

Embedding space



Why?

- Macroeconomics: More work to be done
- That's not what got me interested in embedding space, however.

The problem

1950 First successful kidney transplant

1964 Antirejection therapy

1984 U.S. Congress establishes Organ Procurement and Transplantation Network (OPTN)

2007 Centers for Medicare and Medicaid Services (CMS) quality standards

Question

Do quality standards reduce patient welfare (benefit)?

Unintended consequences

Welfare reduction from

waitlist culling of higher-risk patients
$\rightarrow$ fewer patients receive transplants

increased steroid use
$\rightarrow$ short-term increase in transplant success
$\rightarrow$ longer-term health costs

Problems

We don't know a transplant center's policies and procedures

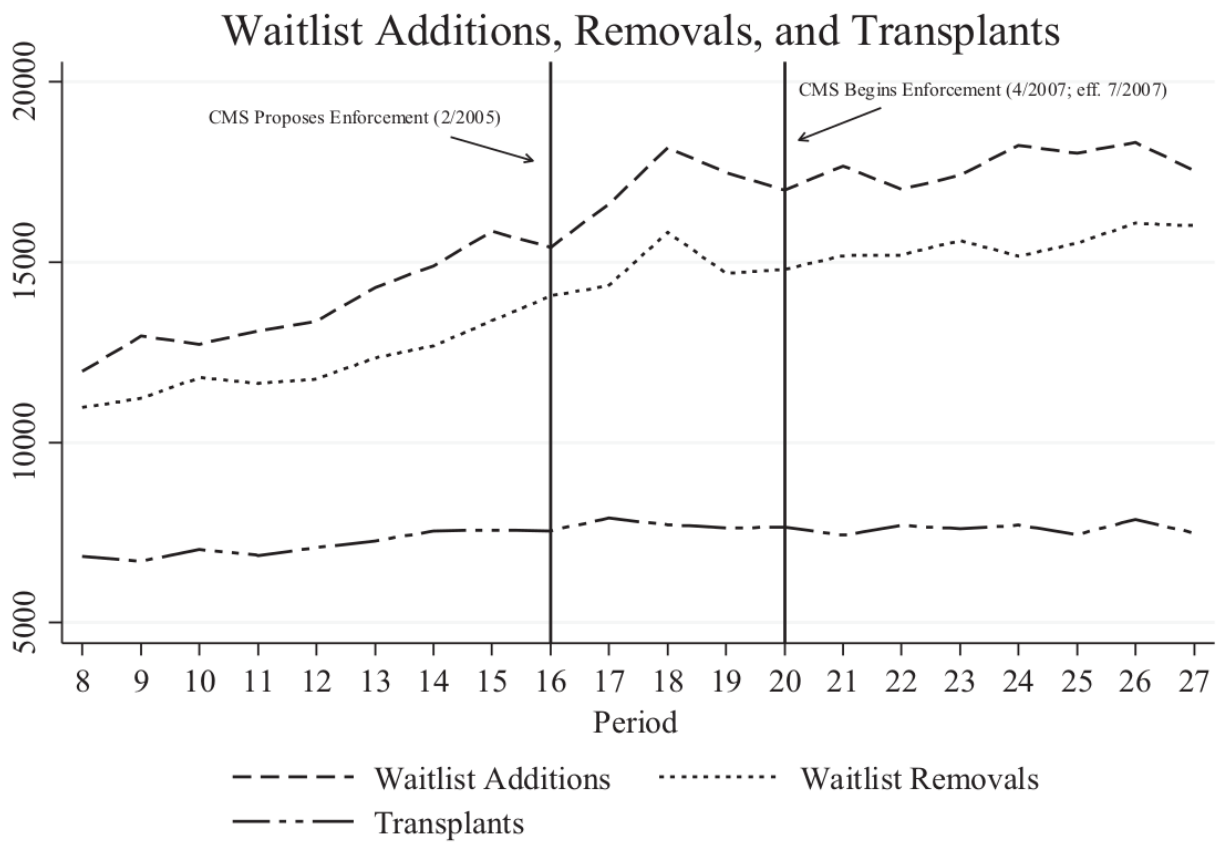
We can see reductions in waitlist sizes over all
$\rightarrow$ Not significant at the individual transplant center level

Problems

We don't k

We car

→ No



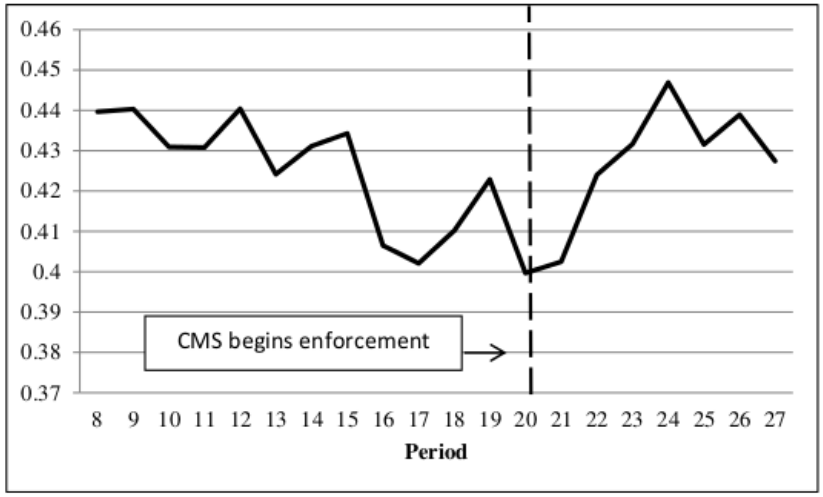
Problems

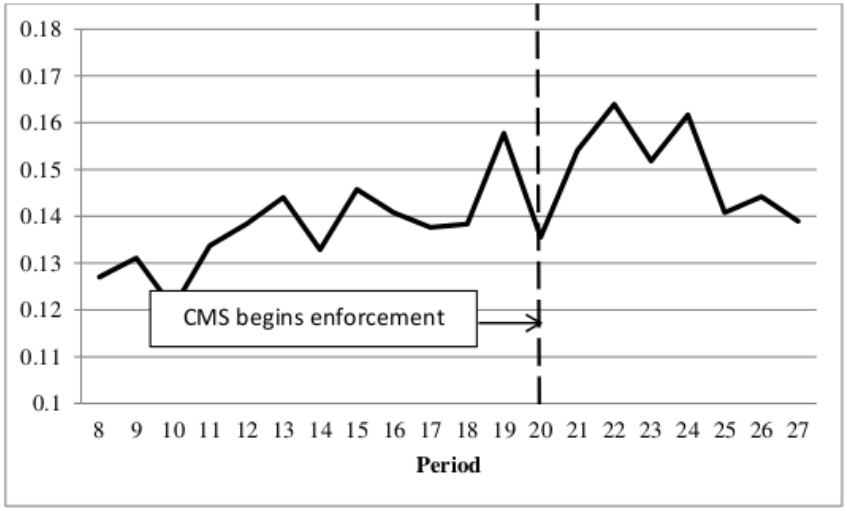
We don't know a transplant center's policies and procedures

- We can see reductions in waitlist sizes over all
  $\rightarrow$ Not significant at the individual transplant center level
- We can see reductions in higher risk transplants over all
  $\rightarrow$ Also not significant at the individual transplant center level

The Problem

Problems





Problems

We don't know a transplant center's policies and procedures

We can see reductions in waitlist sizes over all
$\rightarrow$ Not significant at the individual transplant center level

We can see reductions in higher risk transplants over all
$\rightarrow$ Also not significant at the individual transplant center level

We do have steroid use per transplant (roll up to transplant center)
$\rightarrow$ Still not significant at the individual transplant center level

Problems

We don't k

We car

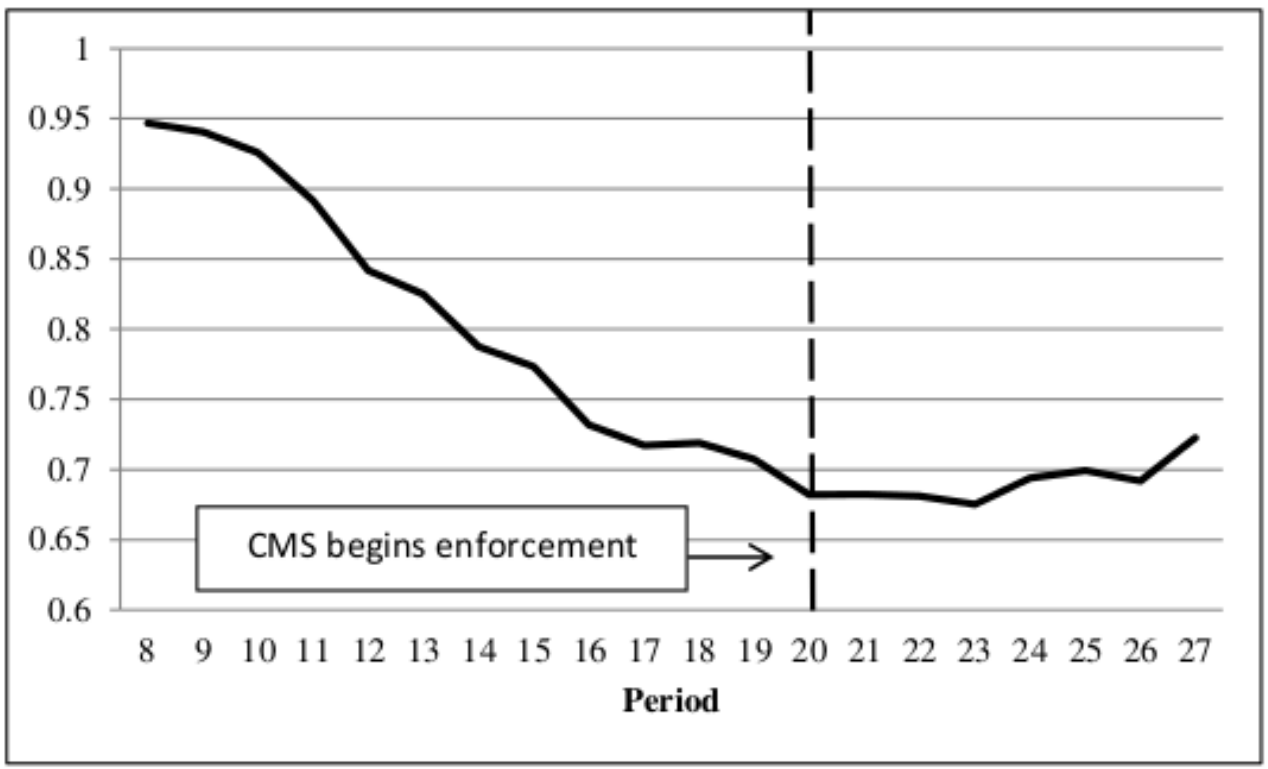
→ No

We car

→ Als

We do

→ Still



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level

t center)

evel

Transplant risk factors

Risk factors known about transplants

Available throughout the data

- Age
- Antigen mismatch: A, B, DR, HLA

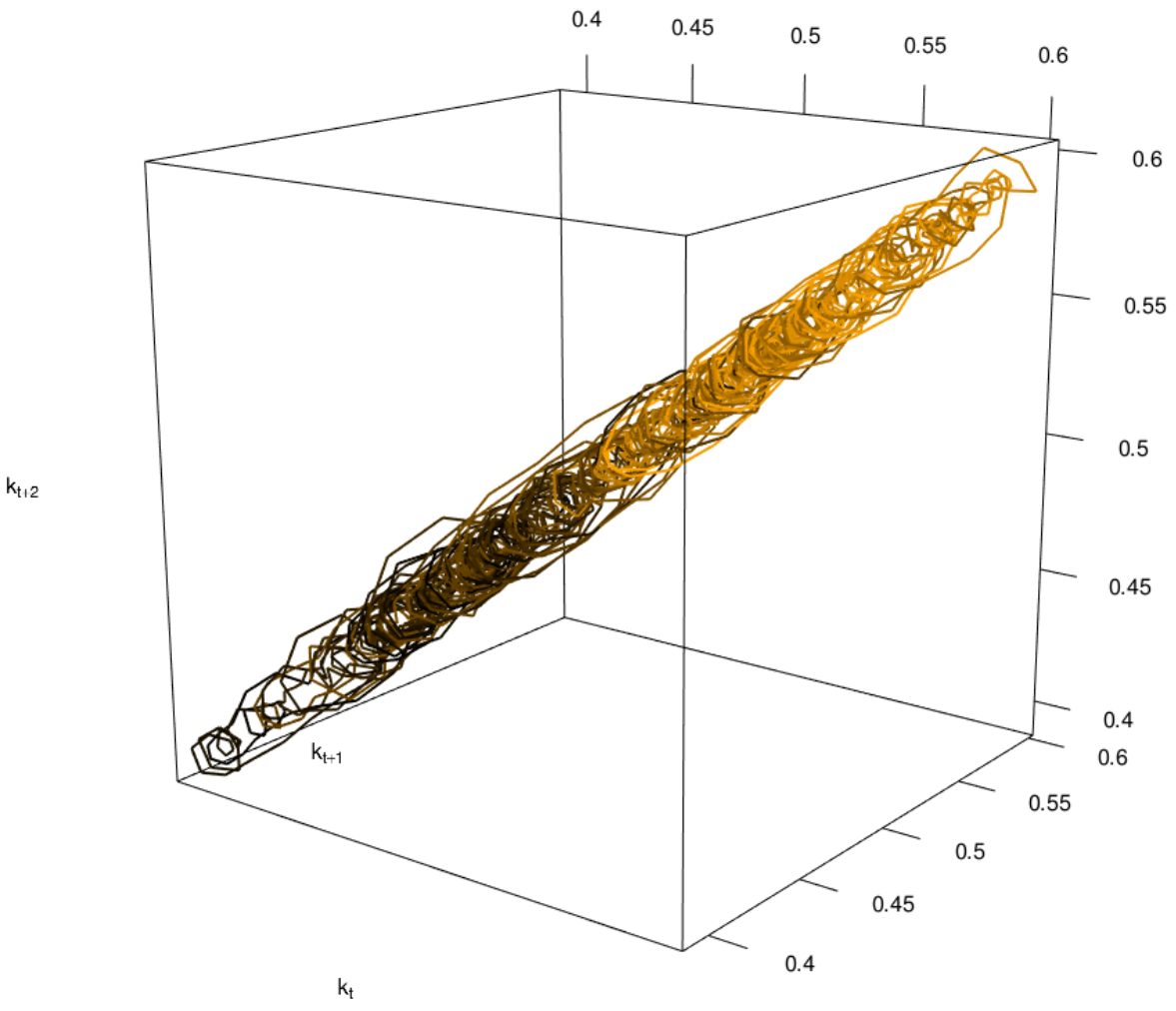
Only available in more recent data

- BMI, ulcers, vascular disease

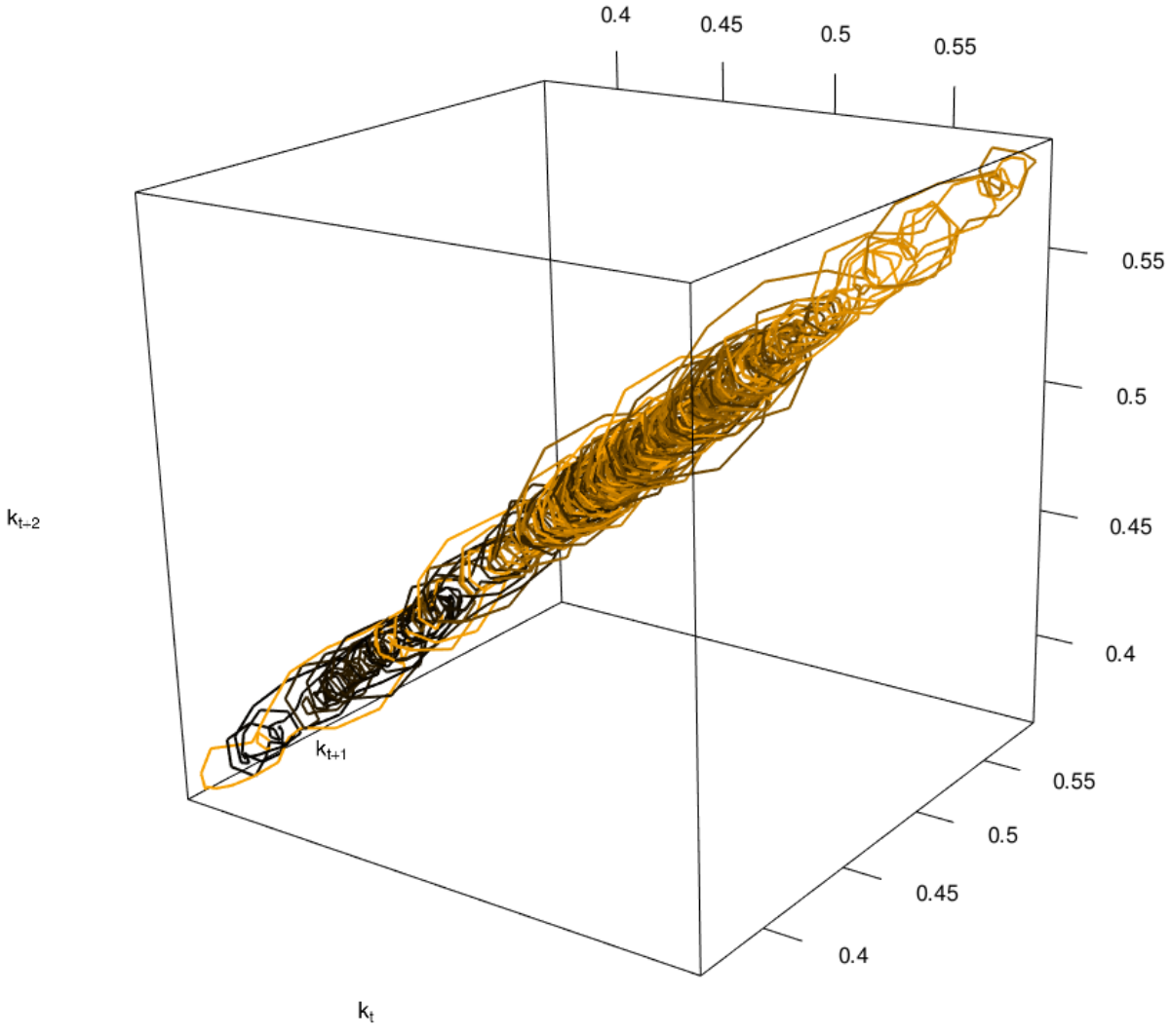
Vector length in 5 dimensional risk space

$$d_t = \sqrt{Age_t^2 + Amis_t^2 + Bmis_t^2 + DRmis_t^2 + HLAmis_t^2}$$

Transplant center embedding space



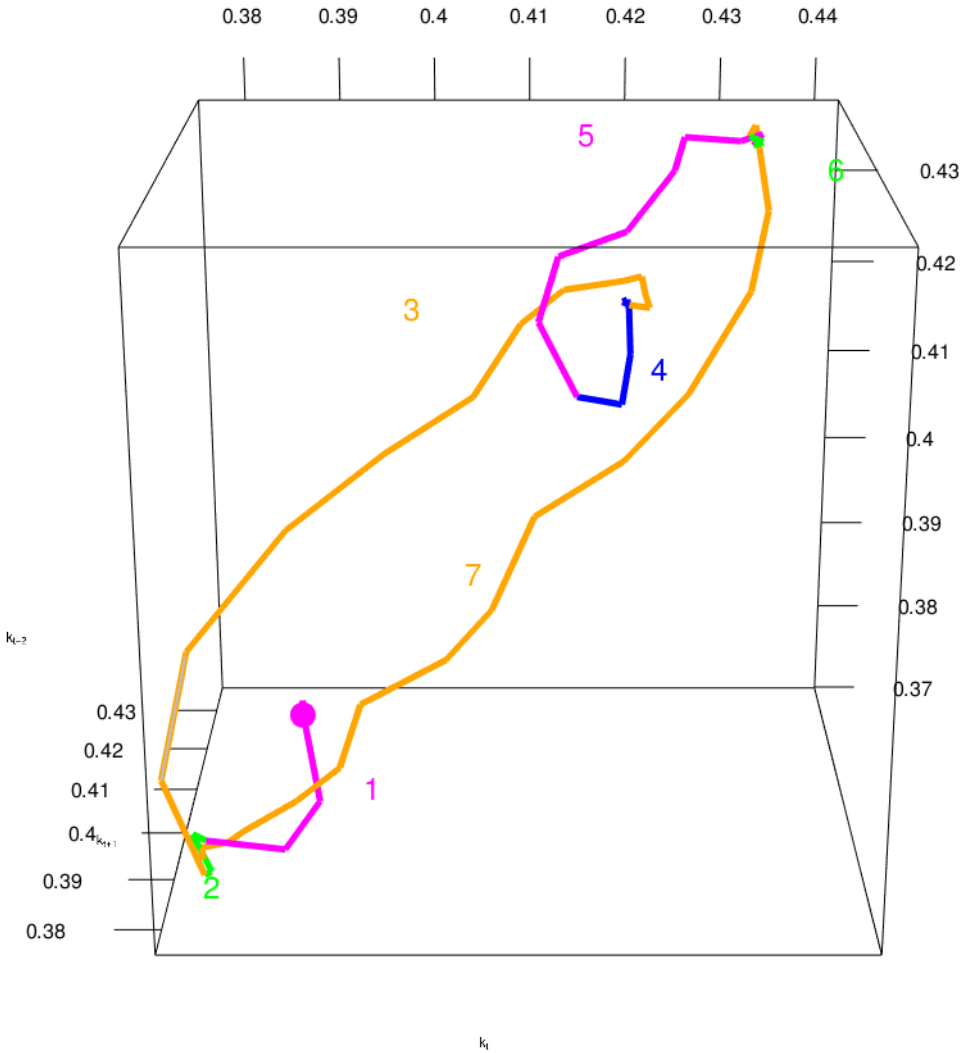
Transplant center embedding space



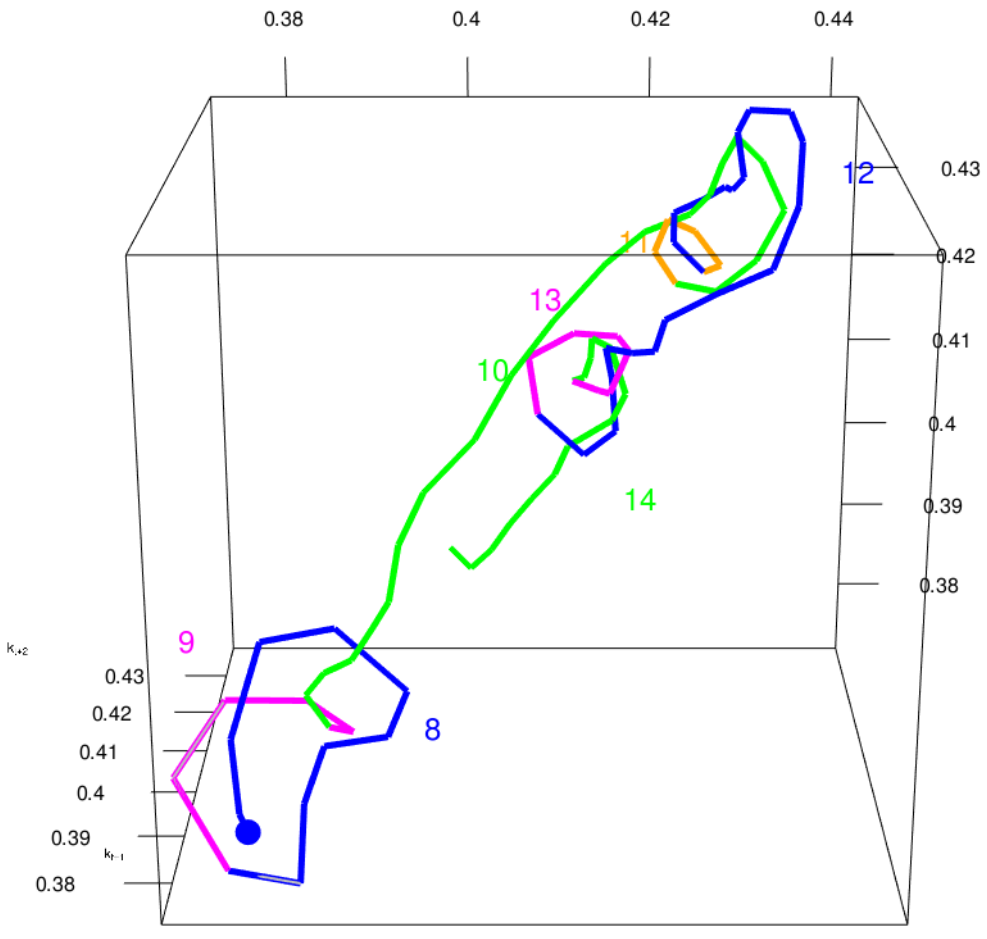
Transplant center embedding space

Movie Time!

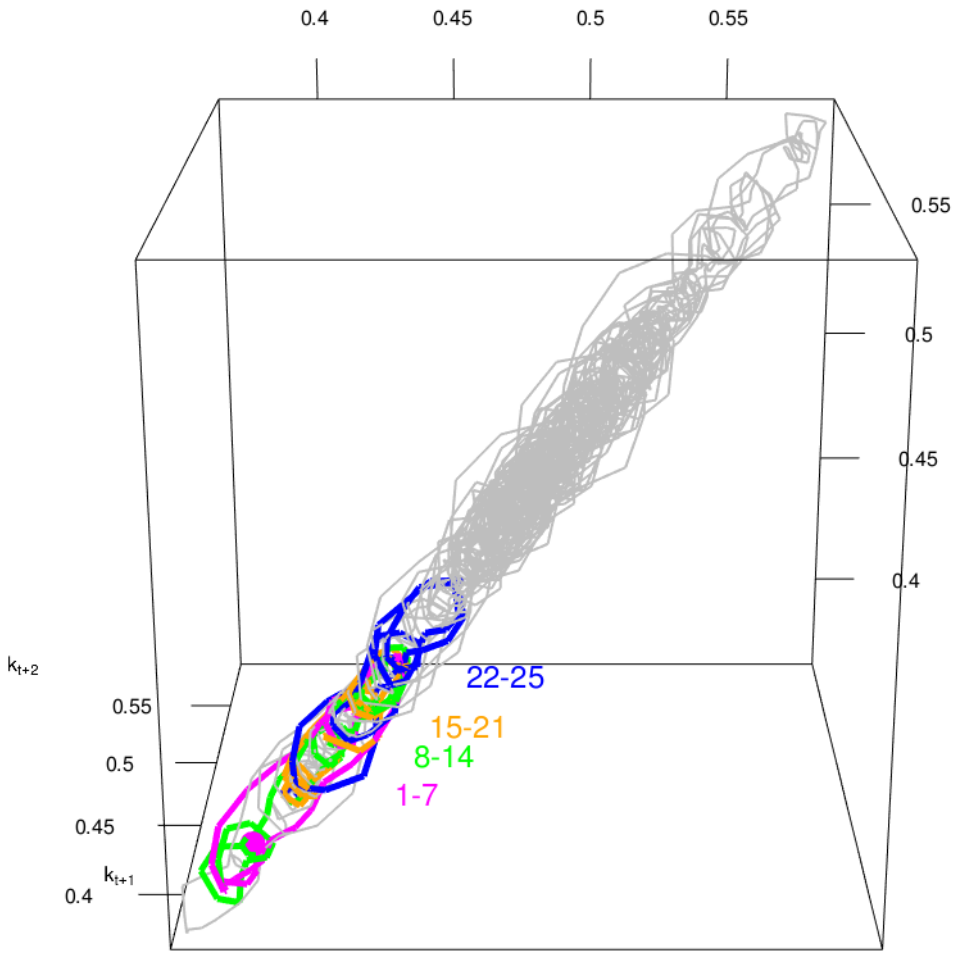
First seven orbits



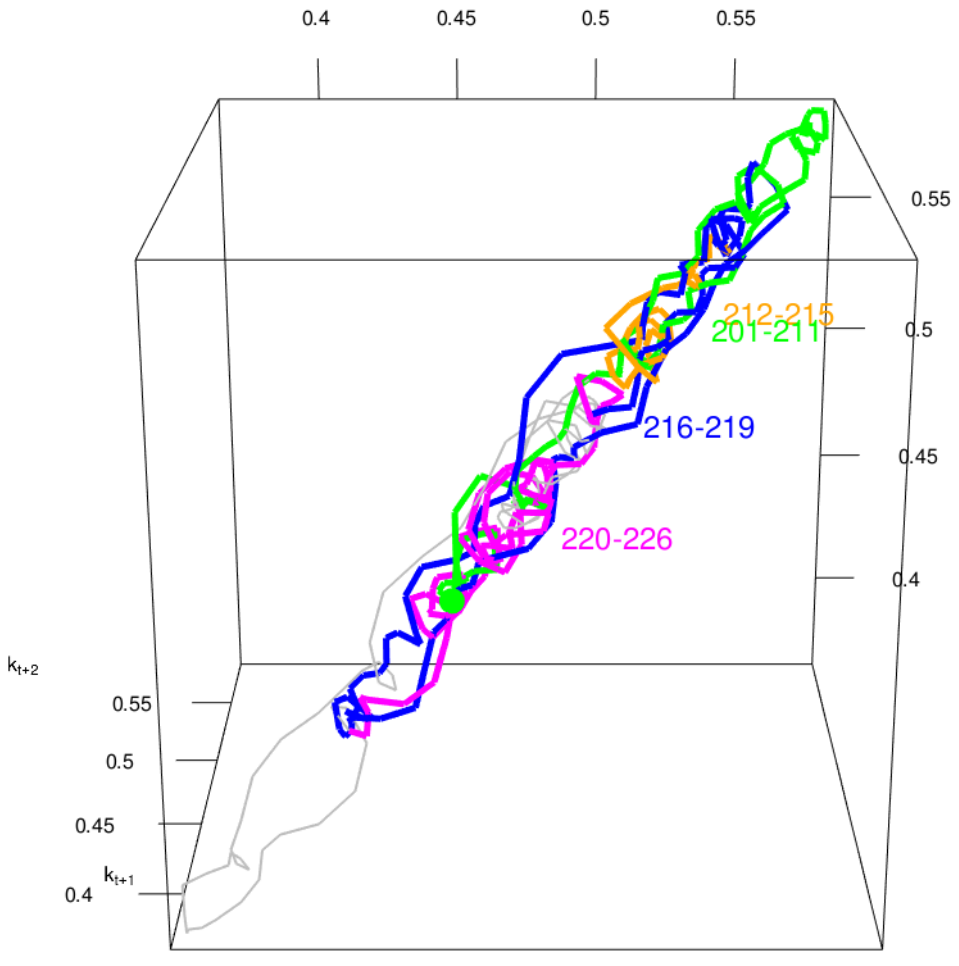
Orbits 8 through 14



Orbits 1 through 25



Orbits 201 through 226



The transplant question

The problems with traditional approaches (econometrics)

- Separation of the mean from dynamics requires reducible periodocity
- Analysis of the dynamics requires constant dimensionality

CMS Compliance

Centers for Medicare and Medicade Services CMS compliance is

- 6 different statistical assessments of one-year survival
- 30 month overlapping windows
- 6 months apart

That is, a graft failure will not affect assessment until 18 months later.

However,

a transplant center **should** know as graft failures increase.

About this transplant center

This transplant center

- First borderline noncompliance
  - does not significantly change risk portfolio
  - does not resort to higher steroid use
- Second borderline noncompliance
  - higher variance in risk portfolio
  - closes after two periods of noncompliance

Future transplant work

299 transplant centers to go!

Two measures of complexity

Another measure of complexity

Lyapunov exponent

In mathematics the **Lyapunov exponent** or **Lyapunov characteristic exponent** of a dynamical system is a quantity that characterizes the rate of separation of infinitesimally close trajectories.

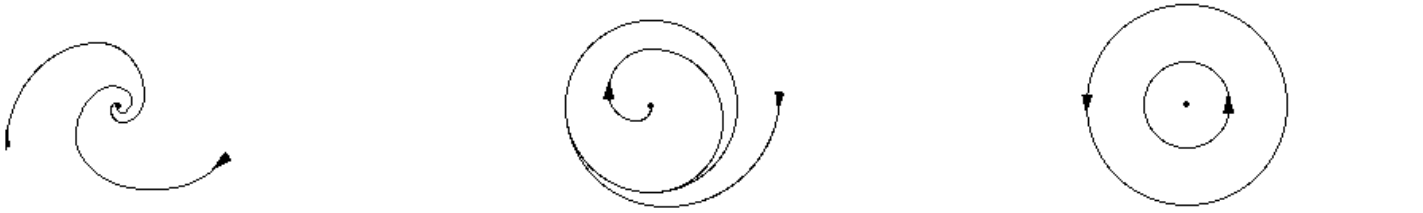
-Wikipedia

Lyapunov exponent

Maximum Lyapunov exponent (MLE) λ

- The rate at which phase-space trajectories diverge
  - $\lambda > 0$ chaotic
  - $\lambda \approx 0$ periodic
  - $\lambda < 0$ mean-reverting or critically damped

Some orbits with their Lyapunov exponents

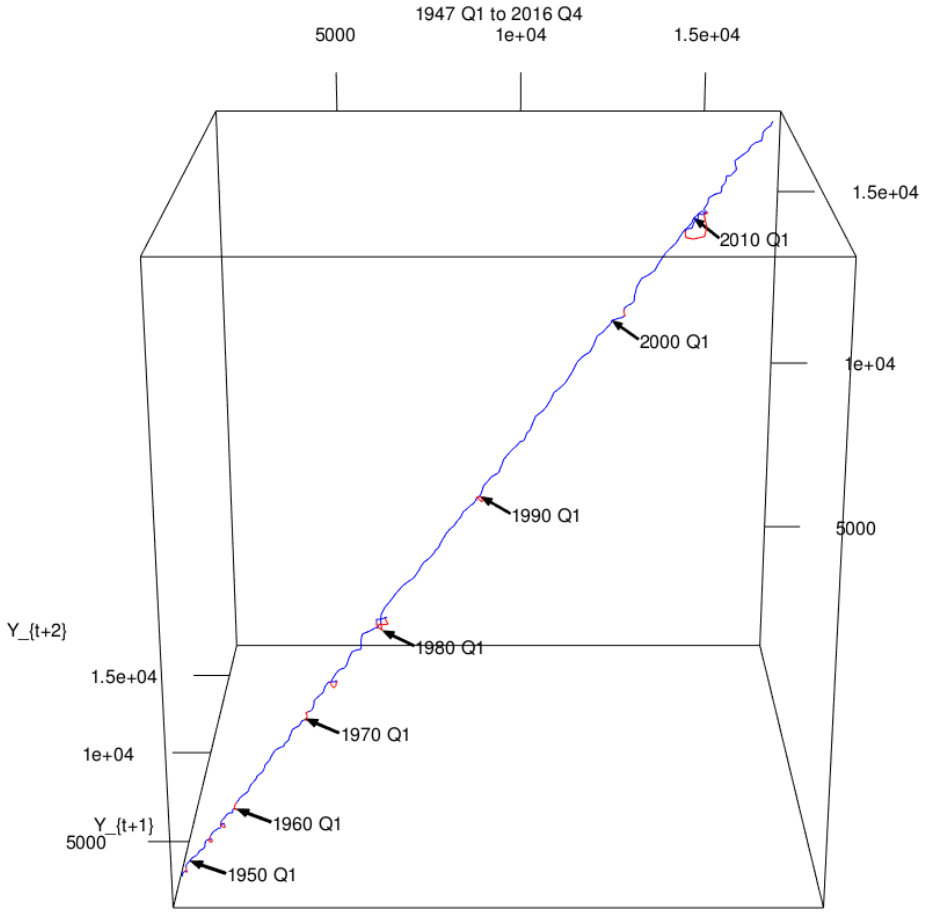


Dissipative, attracting fixed point, $\lambda < 0$

Dissipative, (attracting orbit), $\lambda < 0$

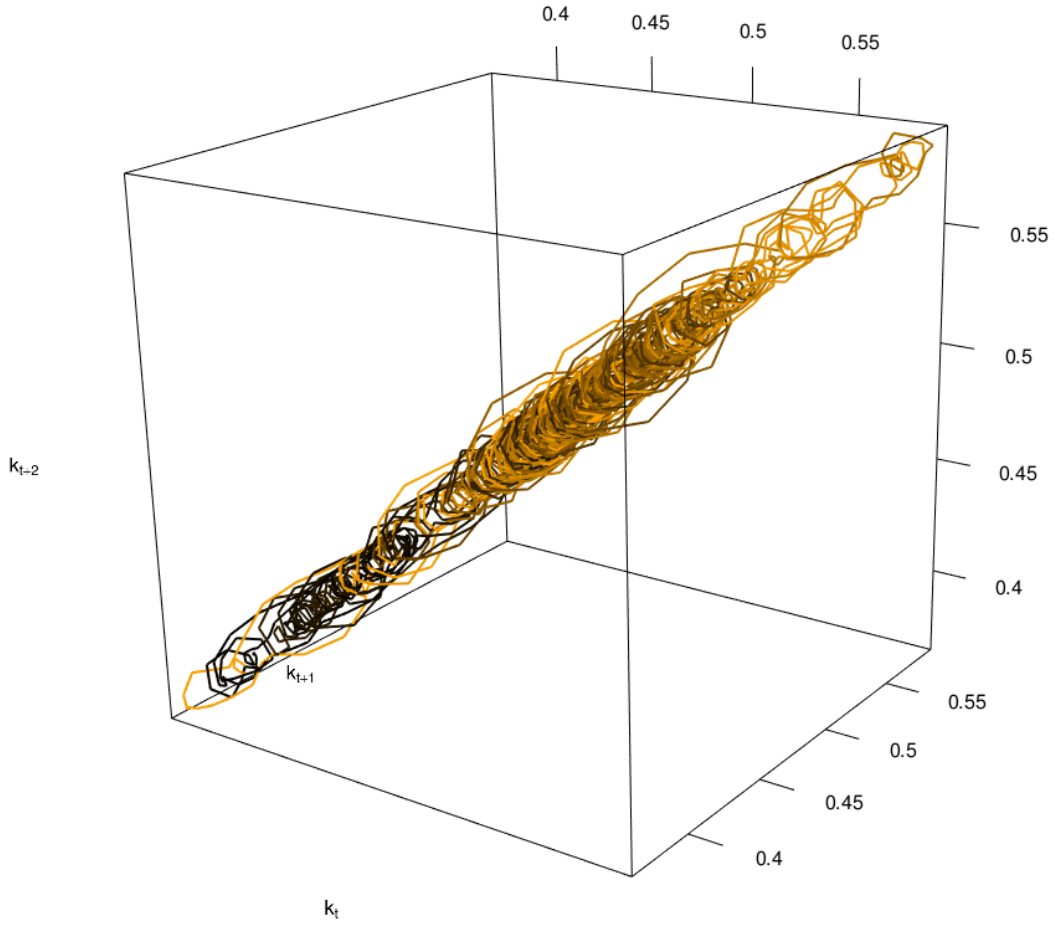
Conservative, neutral fixed point & closed orbits), $\lambda = 0$

MLE for GDP



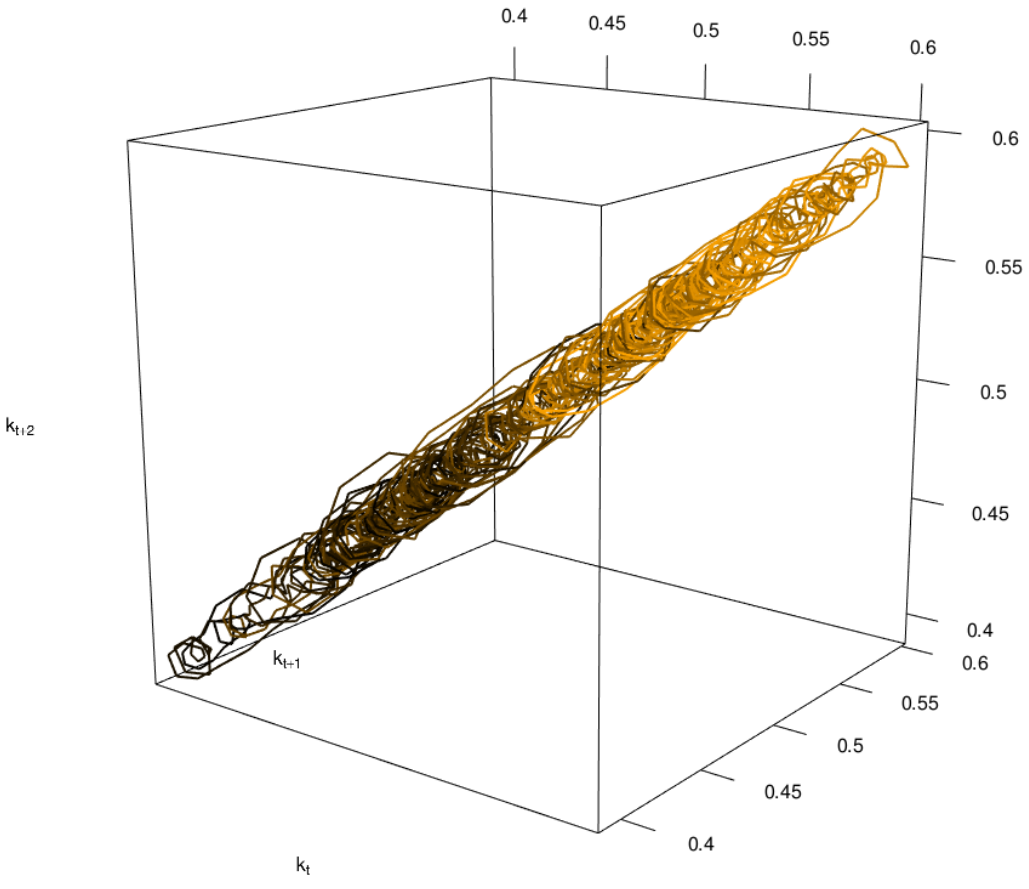
MLE = -0.016

MLE for transplant center



MLE = 0.0094

MLE for other transplant center



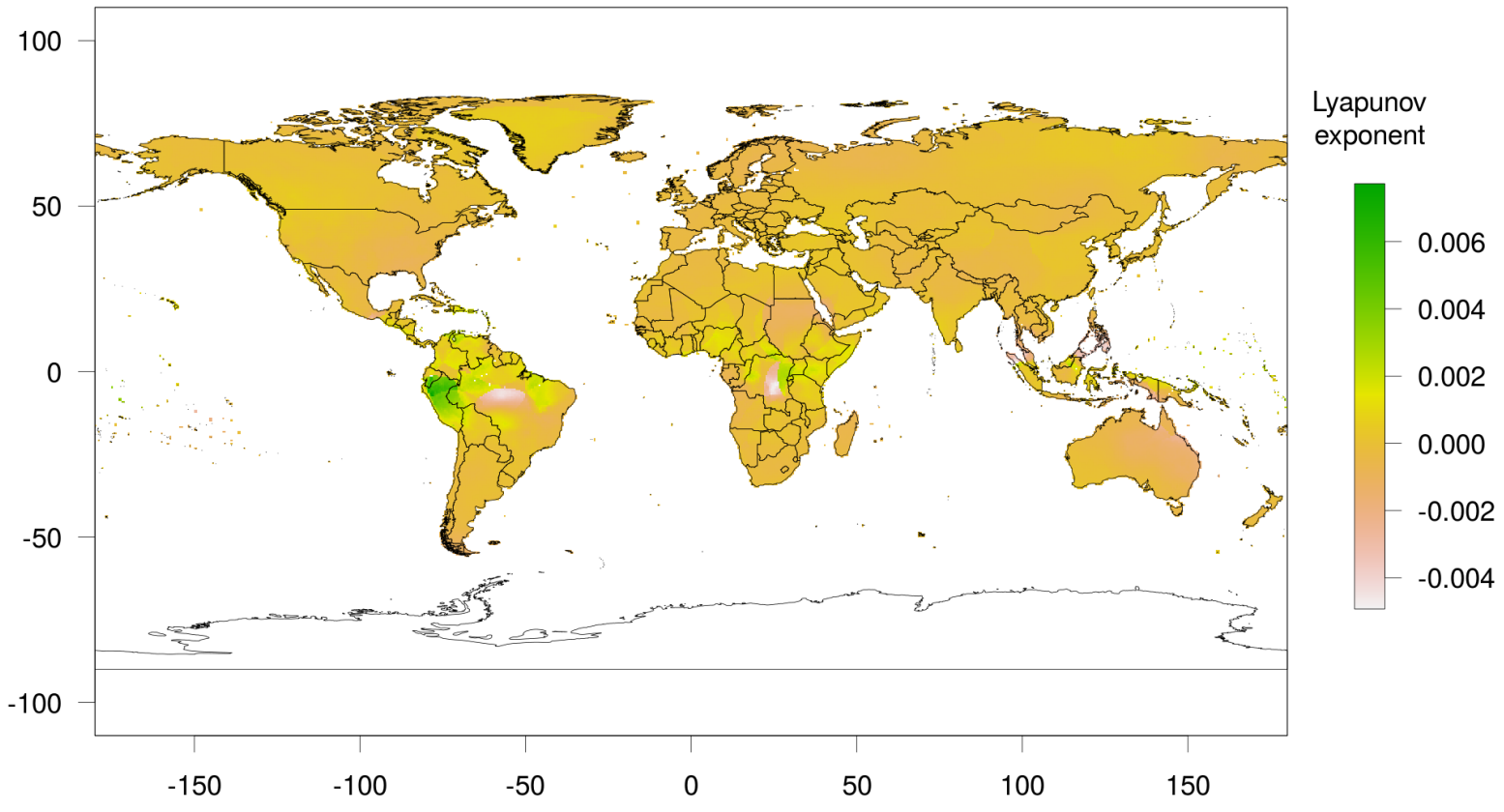
MLE = 0.0016

Lyapunov exponent

How's the weather?

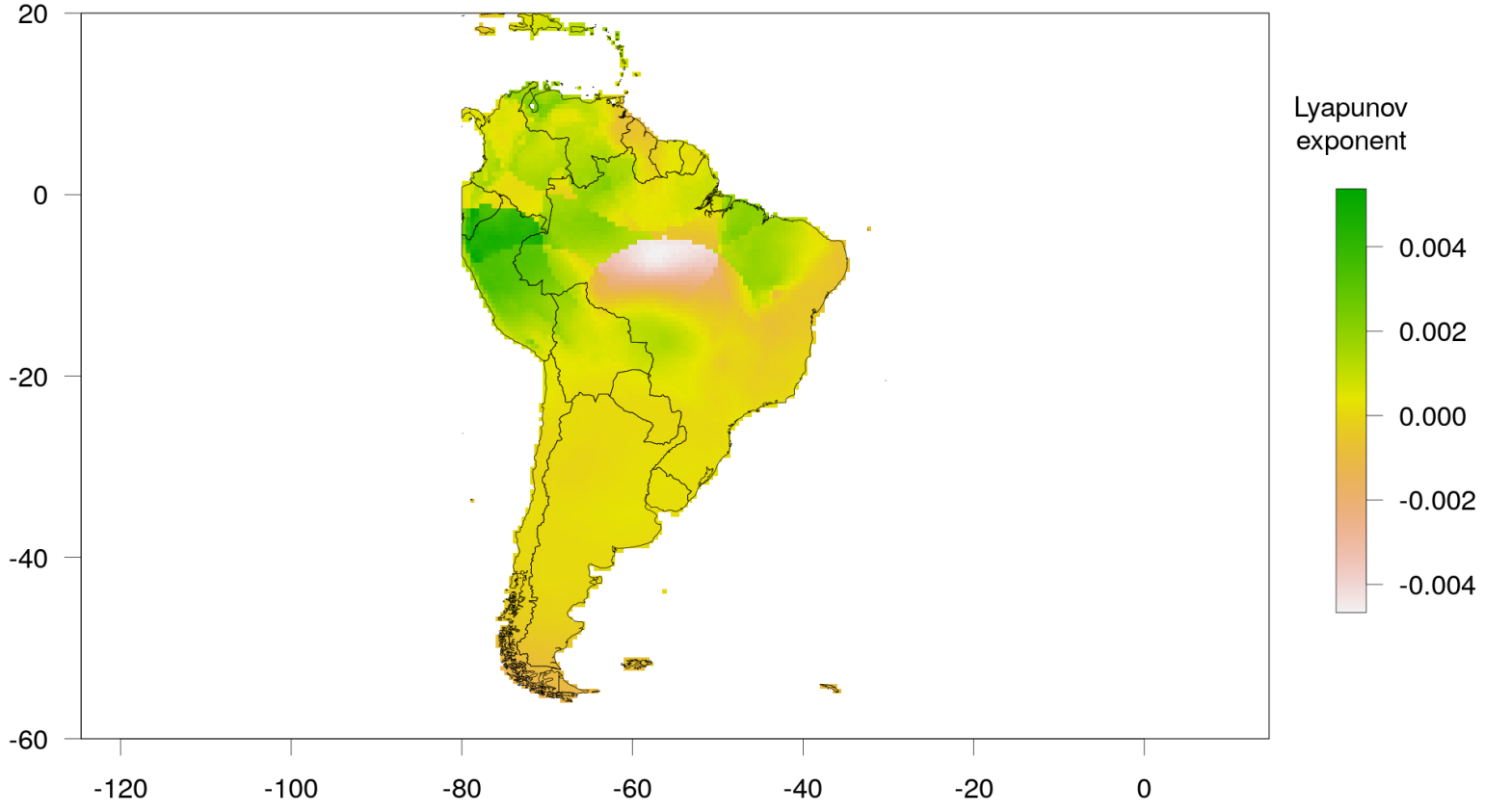
Global temperature Lyapunov exponent

MLE for temperature monthly from 1 Jan 1901 to 16 Dec 2016



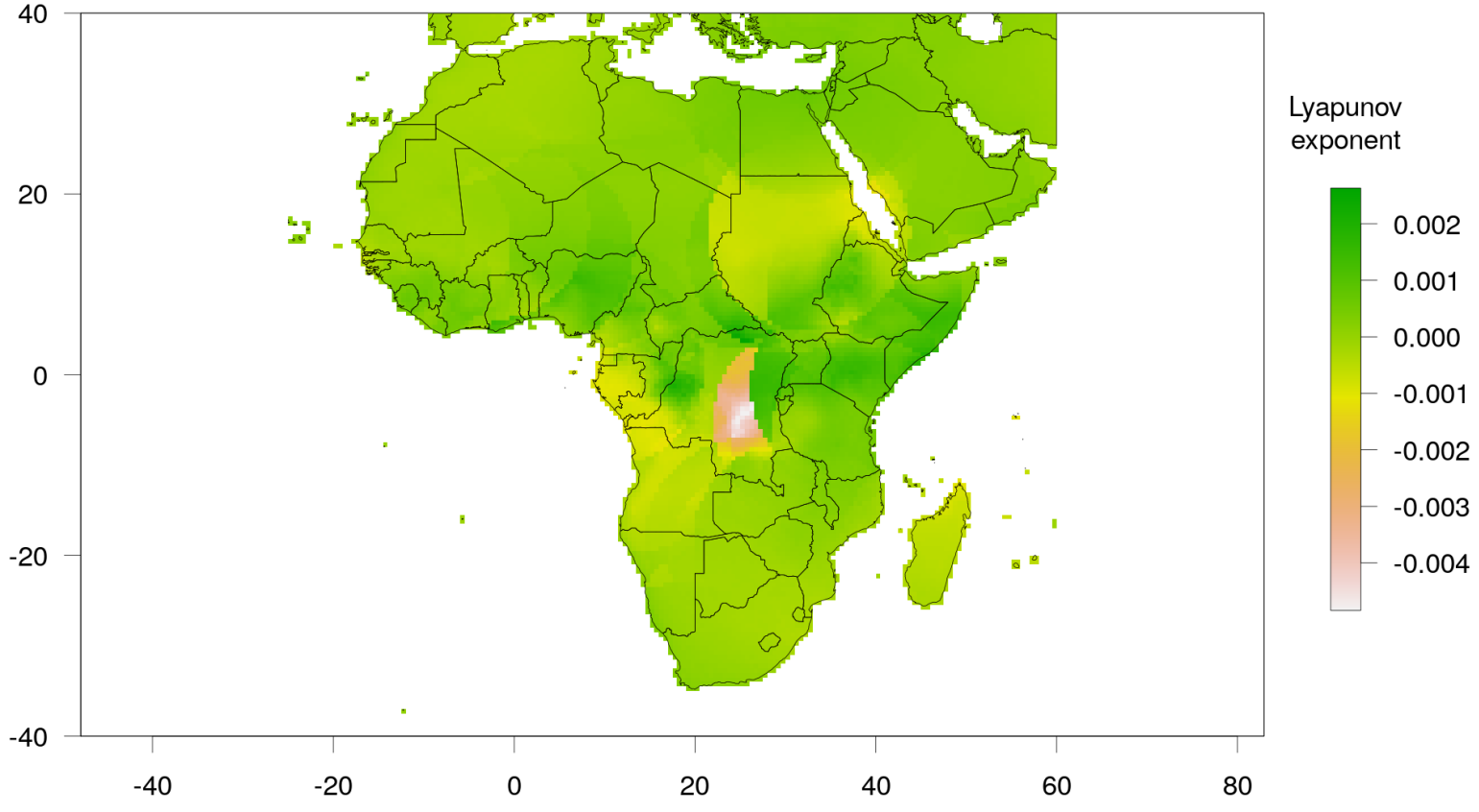
South America temperature Lyapunov exponent

South America, temperature, 1 Jan 1901 to 16 Dec 2016



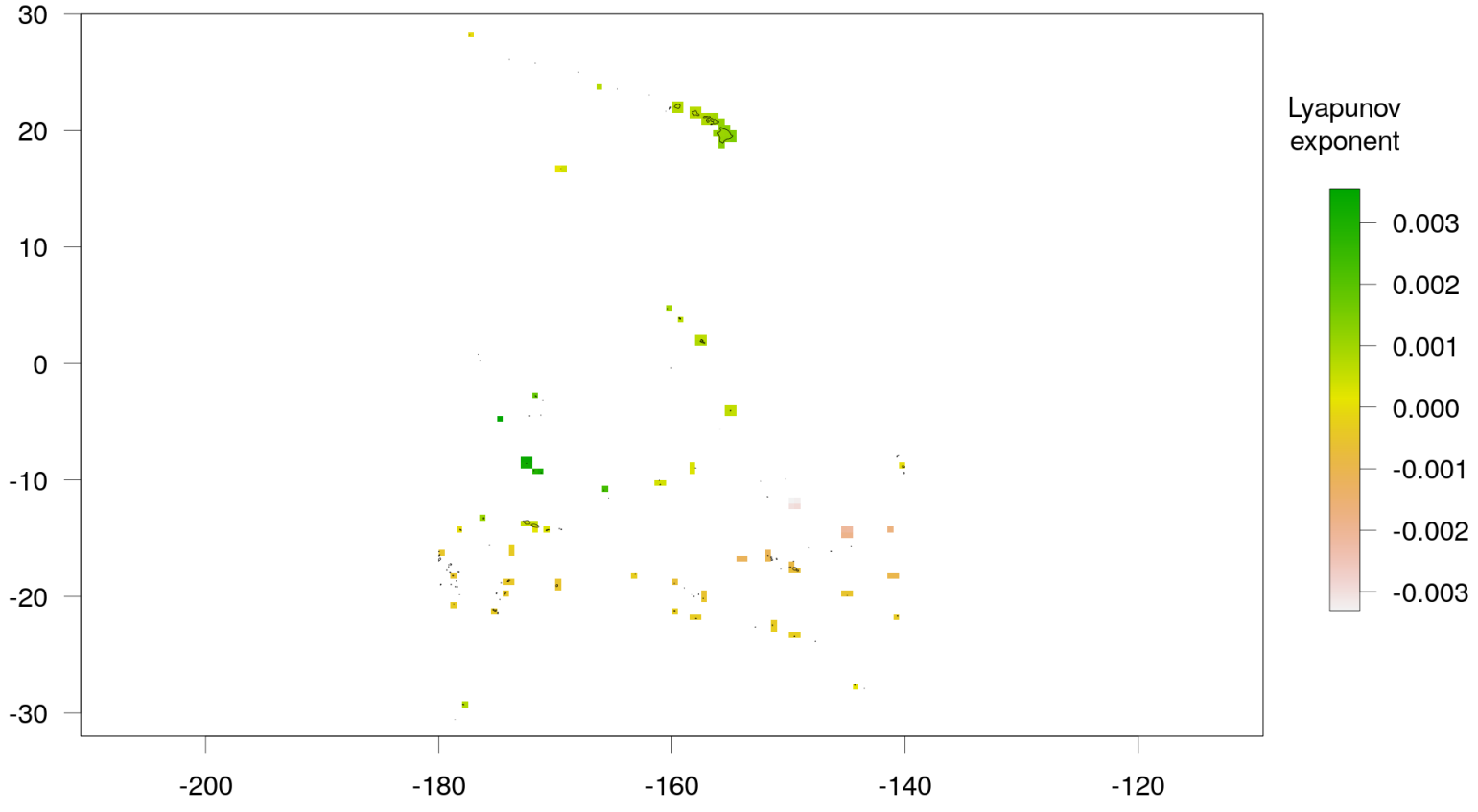
Africa temperature Lyapunov exponent

Africa, temperature, 1 Jan 1901 to 16 Dec 2016



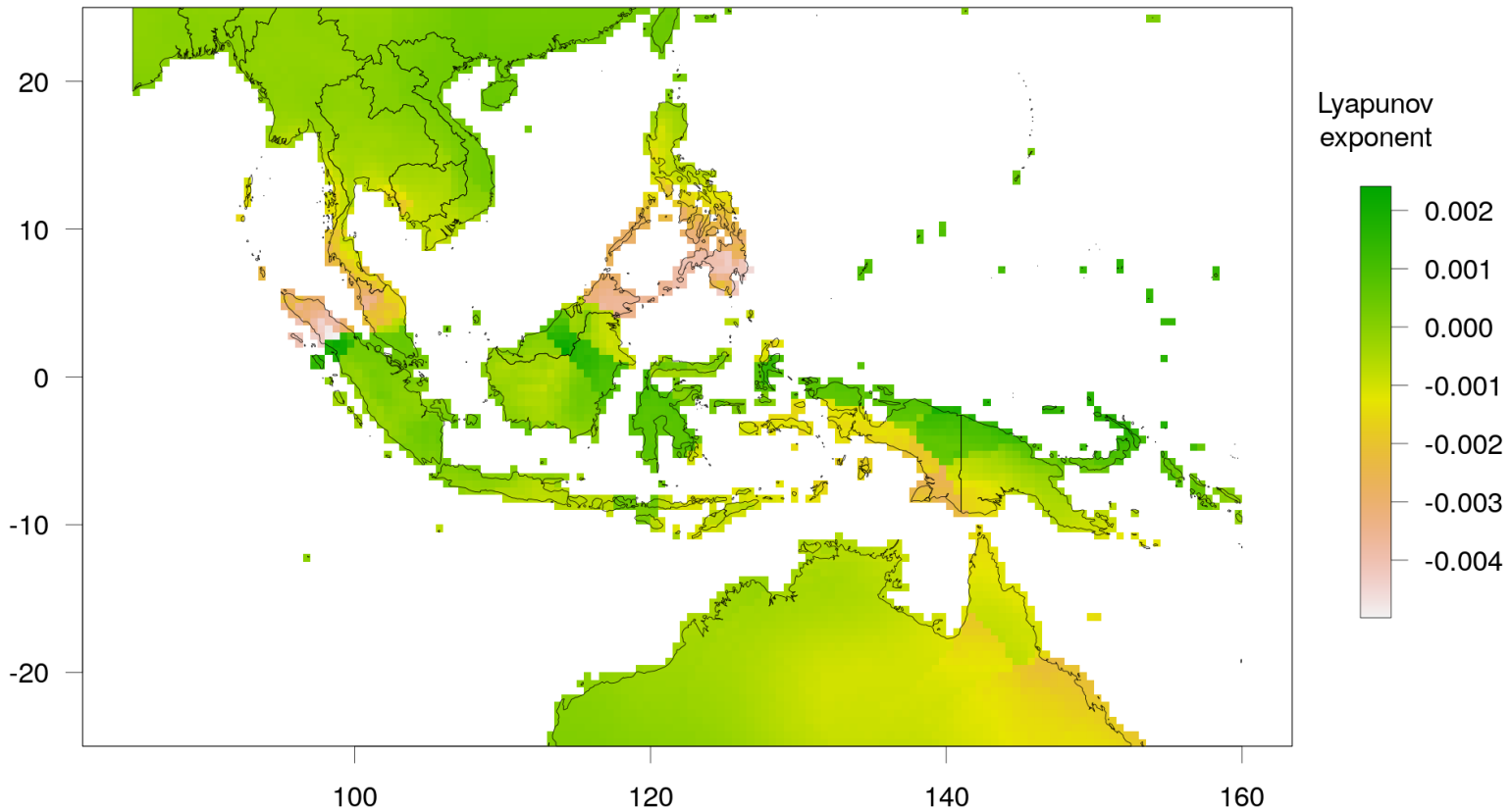
Pacific temperature Lyapunov exponent

Pacific, temperature, 1 Jan 1901 to 16 Dec 2016



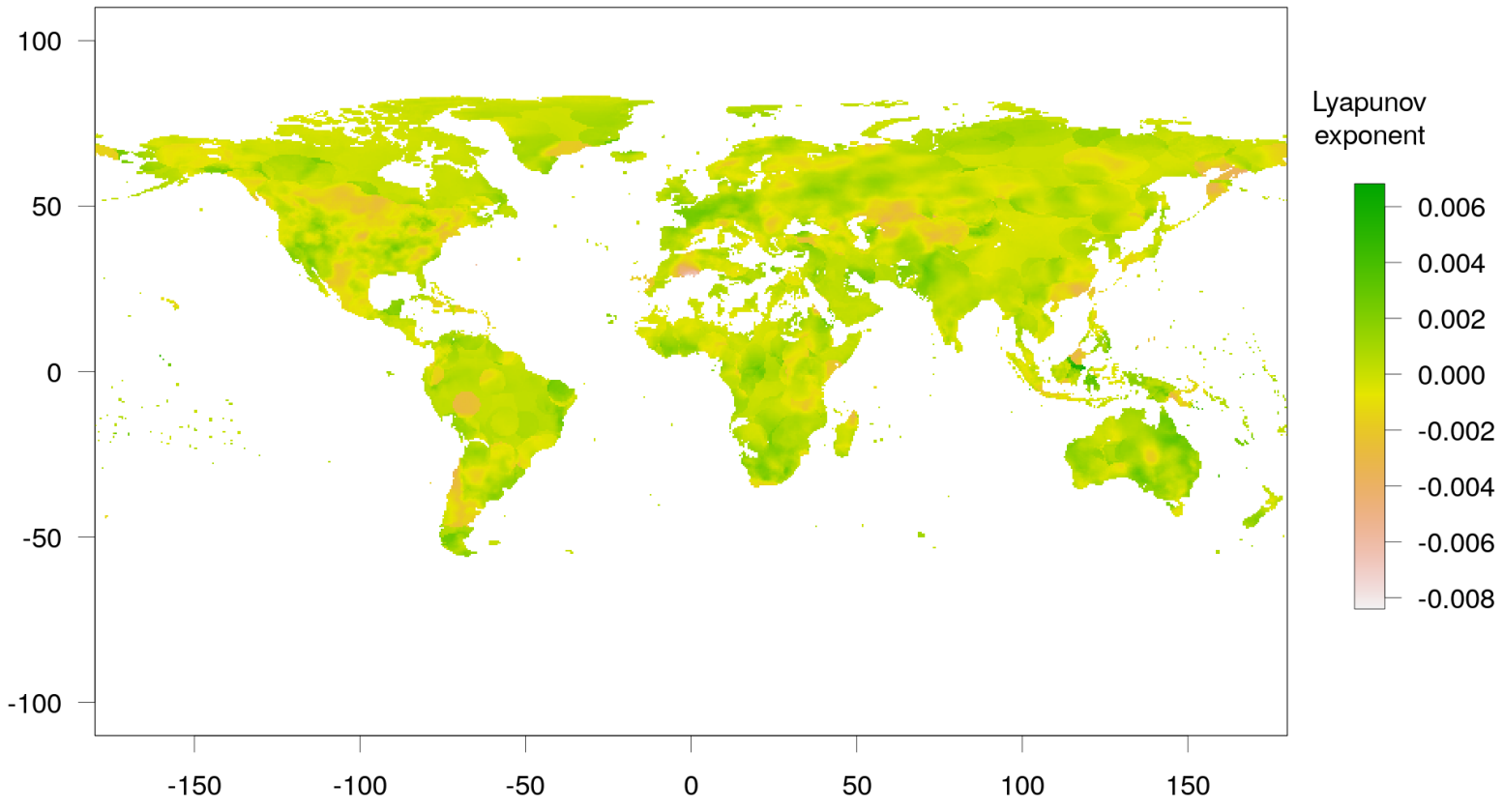
Southeast Asia temperature Lyapunov exponent

Southeast Asia, temperature, 1 Jan 1901 to 16 Dec 2016



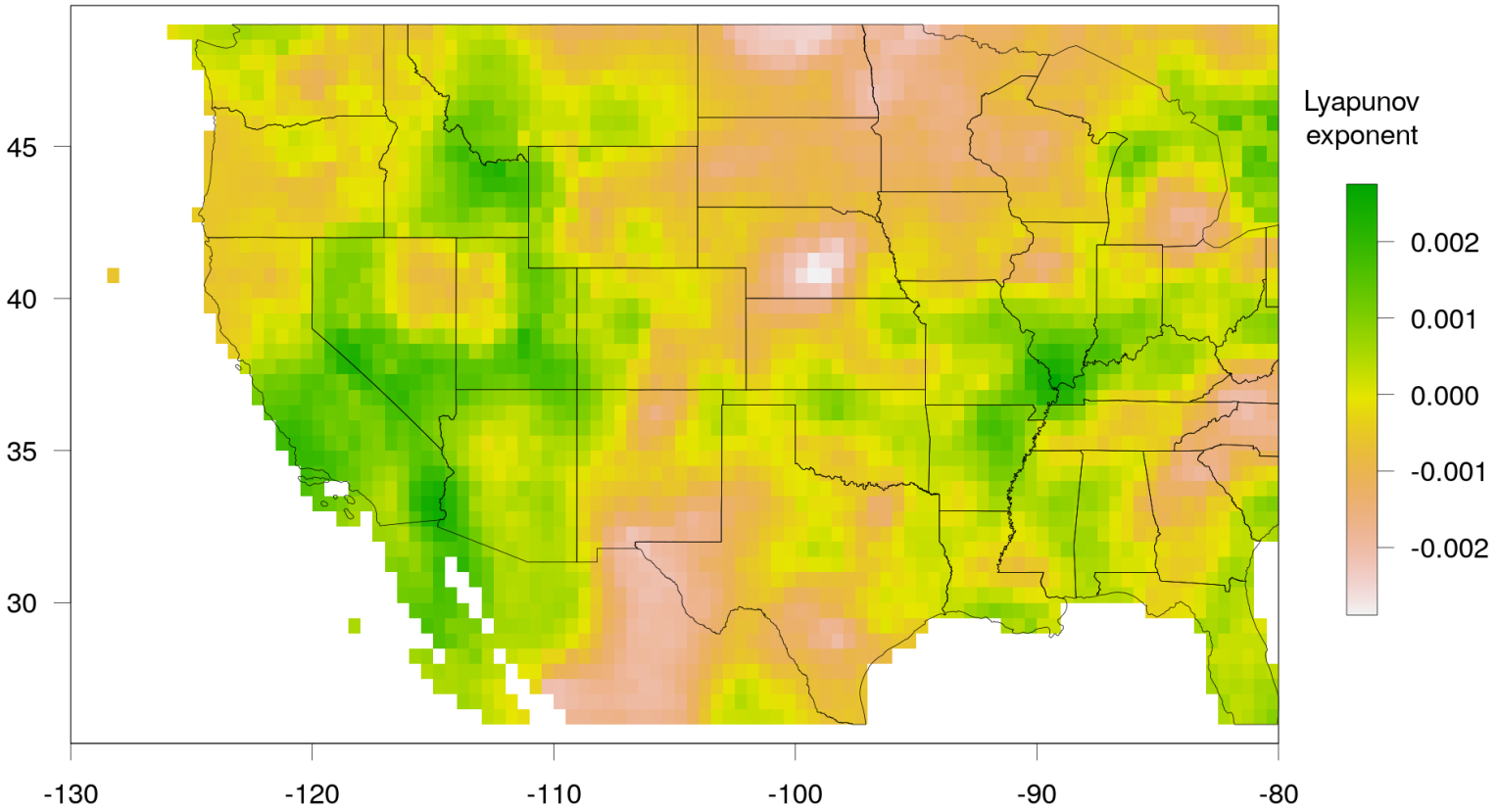
Global precipitation Lyapunov exponent

MLE for precipitation monthly from 1 Jan 1901 to 16 Dec 2016



Southwest precipitation Lyapunov exponent

Southwest US, precipitation, 1 Jan 1901 to 16 Dec 2016



Next step

Does MLE correspond to other trends?

- variability and agriculture
- variability and health
- variability and violence

The End

Thank you!